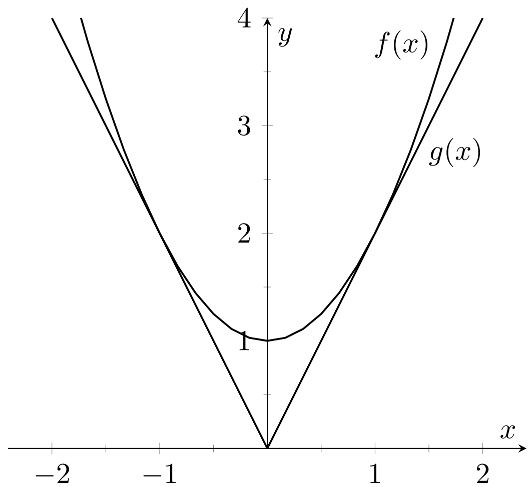
Math 3B **Quiz 2**

Name:____________________________________ Section Time:____________

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know or what you would do will get you partial credit!

1. Consider the following functions and their accompanying graphs.

$$f(x) = x^2 + 1, \qquad g(x) = |2x|, \qquad -\infty < x < \infty$$



(a) Find the points of intersection of $f(x)$ and $g(x)$ algebraically. Express each point of intersection as an ordered pair (x, y).

(b) Find the area bounded by the curves $f(x)$ and $g(x)$.

(a) Our first task is to find the points of intersection. This can be done by setting

$$f(x) = g(x).$$

Unfortunately, it's somewhat difficult to work with the absolute value function, so we may consider $g(x)$ as a piecewise function:

$$g(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x \leq 0 \end{cases}.$$

To find the points of intersection, we first suppose $x \geq 0$. We then set

$$x^2 + 1 = 2x$$

or

$$x^2 - 2x + 1 = 0.$$

Since $x^2 - 2x + 1 = (x-1)^2$, we find the single root $x = 1$. This gives us the intersection point $(1, f(1)) = (1, g(1)) = (1, 2)$. Checking for other intersection points, we suppose $x < 0$. This prompts us to set

$$x^2 + 1 = -2x$$

or

$$x^2 + 2x + 1 = 0.$$

Since $x^2 + 2x + 1 = (x+1)^2$, we find another single root $x = -1$, which gives us our other intersection point $(-1, f(-1)) = (-1, g(-1)) = (-1, 2)$. We conclude that our two points of intersection are the points

$$(1, 2) \qquad \text{and} \qquad (-1, 2).$$

(b) To find the area bounded by these two graphs, we must compute

$$\int_{-1}^{1} f(x) - g(x)\, dx = \int_{-1}^{1} x^2 + 1 - |2x|\ dx.$$

Once again, this integral will have to be split between the part where $x \geq 0$ and the part where $x < 0$:

$$\begin{aligned}
\int_{-1}^{1} x^2 + 1 - |2x|\ dx &= \int_{-1}^{0} x^2 + 1 - (-2x)\, dx + \int_{0}^{1} x^2 + 1 - 2x\, dx \\
&= \frac{1}{3}x^3 + x^2 + x\Big|_{-1}^{0} \quad + \quad \frac{1}{3}x^3 - x^2 + x\Big|_{0}^{1} \\
&= 0 - \left(-\frac{1}{3} + 1 - 1\right) + \left(\left(\frac{1}{3} - 1 + 1\right) - 0\right) \\
&= \frac{2}{3}.
\end{aligned}$$

Note that by symmetry, we could have also computed this integral as

$$\begin{aligned}
\int_{-1}^{1} x^2 + 1 - |2x|\ dx &= 2\int_{0}^{1} x^2 + 1 - 2x\, dx \\
&= 2\left(\frac{1}{3}x^3 - x^2 + x\right)\Big|_{0}^{1} \\
&= 2\left(\left(\frac{1}{3} - 1 + 1\right) - 0\right) \\
&= \frac{2}{3}.
\end{aligned}$$